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Propagation of travelling waves in sub-excitable systems driven by noise and periodic forcing*

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Abstract. It has been reported that traveling waves propagate periodically and stably in sub-excitable systems driven by noise [Phys. Rev. Lett. **88**, 138301 (2002)]. As a further investigation, here we observe different types of traveling waves under different noises and periodic forces, using a simplified Oregonator model. Depending on different noises and periodic forces, we have observed different types of wave propagation (or their disappearance). Moreover, reversal phenomena are observed in this system based on the numerical experiments in the one-dimensional space. We explain this as an effect of periodic forces. Thus, we give qualitative explanations for how stable reversal phenomena appear, which seem to arise from the mixing function of the periodic force and the noise. The output period and three velocities (normal, positive and negative) of the travelling waves are defined and their relationship with the periodic forces, along with the types of waves, are also studied in sub-excitable system under a fixed noise intensity.

PACS. 82.40.Ck Pattern formation in reactions with diffusion, flow and heat transfer – 05.40.Ca Noise – 47.54.-r Pattern selection; pattern formation – 83.60.Np Effects of electric and magnetic fields

1 Introduction

The effects of noise on nonlinear systems are the subject of intense experimental and theoretical investigations. Noise can induce transition [1,2], bifurcations [3], and stochastic resonance [4–7]. Notably, in reference [7] the synchronization of spatiotemporal patterns was observed in an excitable medium via the numerical modelling. Moreover noise can enhance propagation in arrays of coupled bistable oscillators [8–11]. In an excitable system, an external periodic forcing can dramatically change its behavior. As reported previously phase locking, quasi-periodicity, period doubling, and chaos were observed [12]. The temporal evolution of the concentration patterns has been modeled by partial differential reaction-diffusion equations. Such models include oscillatory, excitable or bistable systems with either none, one or two linearly stable homogeneous states [13,14]. It is also well known that in sub-excitable systems noise can also induce travelling waves [15], drive avalanche behavior [16], and sustain pulsating patterns and global oscillations [17]. Sub-excitable systems under noises and periodic forcing are able to send out travelling and spiral waves. The Belousov-Zhabotinsky (BZ) reaction [18,19] is a popular symbol in the nonlinear dynamical realm to study excitable and sub-excitable systems. It has been widely agreed that the noise and periodic forcing play a very important role in wave propagation and stability.

Recently, it was observed that noise can support wave propagation in sub-excitable [15,17,20] systems due to a noise-induced transition [21,22]. In the systems studied, the media are static, and transport is governed by diffusion [23]. However in many situations, the media are not static but subject to a motion. For example, stirred by a flow, or by periodic forcing, convective-like phenomena were observed due to applied electric field in reference [24], which occurs especially in chemical reactions in a fluid environment. In such cases, diffusive transport usually dominates only at small spatial scales while mixing due to the flow is much faster at large scale. In references [25–27], the authors show that in an inhomogeneous self-sustained oscillatory media, an increasing rate of mixing can lead to a transition to a global synchronization of the whole media. Especially, reference [28] showed that the interplay among excitability, noise, diffusion and mixing can generate various pattern formation in a 2D

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FigzHugh-Nagumo (FHN) model subject to advection by a chaotic flow. Here, we research the effect of noise and periodic forcing on sub-excitable systems using the Oregonator model in one dimension, which advances from the BZ reaction. The reversal phenomenon is not observed in any publication literatures the propagation of travelling waves. It is found in this paper, which relates to a new concept. In reference [30], Albanese refers to the reversal concept in wave propagation concerning extraction of information about distant structural features from the measurements of scattered waves, but it is irrelevant to the excitable system.

In our paper, we define that general traveling waves propagate forward in one constant direction and vice versa. Under this definition, we find the reversal phenomena in our numerical simulations. However in our simulation we have discovered that after some time, the waves change propagation direction and turn backward to travel in the opposite direction. The traveling waves propagate forward and backward alternately and periodically. That is called the reversal phenomenon (see the supplementary material on-line movie for this phenomenon, *Movie-0*). In mathematical language, the definition of a "reversal phenomenon" is that at time t, the spatial position of a traveling wave front is at point x. After some time $\Delta t > 0$, the wave front reaches point x again with reversed direction. In fact, this phenomenon was observed in the FHN model by numerical simulations [28]. In this article, our focus is on the effect of periodic forcing and noise on the propagation of traveling waves in sub-excitable systems.

2 Model

Most of the systems we are interested in reside in a d-dimensional world. This means that our variables (fields or concentrations) depend on time and space. In the present paper, the starting deterministic model [20] is based on partial differential equations, and when the randomness is introduced we transform them into stochastic partial differential equation. A representative example is the deterministic reaction diffusion equation,

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = f(\phi(\mathbf{x},t),\mu) + \mathcal{D}\nabla^2\phi(\mathbf{x},t), \tag{1}$$

where $\phi(\mathbf{x},t)$ represent the density of a physical observable, $f(\phi(\mathbf{x},t),\mu)$ is a nonlinear function of the field ϕ and μ denotes the relevant control parameter. The above equation can be made more complicated when considering vector fields, higher-order derivatives, or nonlocal operators. The effect of fluctuations is introduced through a stochastic process or noise $\xi(\mathbf{x},t)$ with well controlled statistical properties. As a result, we expect that the new equation governing our system will have the generic form

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = f(\phi(\mathbf{x},t),\mu) + \mathcal{D}\nabla^2\phi(\mathbf{x},t) + g(\phi)\xi(\mathbf{x},t). \tag{2}$$

We take into account this standard example of stochastic partial differential equation and the two-variable Oregonator model [19,35] that is famous for its convenience for studying study the properties of diffusion-reaction systems. Our modified model adds both noise and periodic forces, which is,

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon}f(u,v) + D_u\nabla^2 u + (D_s\xi(t)+E(t))\frac{\partial u}{\partial x}, \tag{3a}$$

$$\frac{\partial v}{\partial t} = g(u,v) + D_v\nabla^2 v + (D_s\xi(t)+E(t))\frac{\partial v}{\partial x}, \tag{3b}$$

where $f(u,v) = u(1-u) - fv\frac{u-q}{u+q}$, and $g(u,v) = u - v$. ∇^2 is the Laplacian operator in Cartesian coordinates. u and v represent the concentration of $HBrO_2$ and the catalyst $2Ce^{4+}$, respectively. Here, the external electric field can be considered as a spatially uniform electric field, which includes two parts: stochastic forcing, $\xi(t)$ (we use the notation of $\xi(t)$ for irrelevant in space) and the periodic force , $E(t)$. Both of them depend on time t with the form of Gaussian noise and with the periodic function respectively. $\xi(t)$ denotes Gaussian white noise with $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t^{'})\rangle = 2D_s\delta(t-t')$, a typical temporally varied Gaussian white noise and here D_s is the intensity of noise. $E(t)$ is the periodic force, where the sine periodic force is chosen, $E(t) = F\sin(\frac{2\pi}{T_{in}}t)$. F and T_{in} are the intensity of the periodic forcing and the input period, respectively. The effect of electric field is convective-like, as discussed in reference [24]. D_u and D_v are the dimensionless diffusion coefficients of u and v, $D_u = 1.0$, $D_v = 0.6$ [31].

The dynamical system (3) is simulated in one-dimensional space by Euler-Maruyama Method [32] with zero flux boundary conditions on a space length of 500 elements. The space step is $\Delta x = 0.15$ space unit and the time step is $\Delta t = 10^{-3}$ time unit. These parameters are chosen to make the simulation process relatively stable and the information of wave propagation can be relatively homogenous uniform. In order to avoid the simulation going to the negative region of u, we let $u > q$, if $u < q$, $u = q$, according to the method of references [33, 34]. The simulations have been done as follows: we excite 3 elements at the left boundary in the our systems under sub-excitable state in the one-dimensional space, which serves as the wave source. The leapfrog method for the advection terms; an implicit method for the diffusion terms; and a simple explicit Euler method for the reaction terms. Letting $U_j^n \approx u(x_j,t_n)$ and $V_j^n \approx v(x_j,t_n)$, where $x_{j+1} - x_j = \Delta x$ and $t^{n+1} - t^n = \Delta t$, results in the following discretised system:

$$\frac{U_j^{n+1}-U_j^n}{\Delta t} = \frac{1}{\varepsilon}f(U_j^n,V_j^n) + D_u\left[\frac{U_{j+1}^{n+1}-2U_j^{n+1}+U_{j-1}^{n+1}}{(\Delta x)^2}\right] + (D_s\xi(t)+E(t))\left[\frac{U_{j+1}^n-U_{j-1}^n}{2\Delta x}\right], \tag{4}$$

$$\frac{V_j^{n+1}-V_j^n}{\Delta t} = g(U_j^n,V_j^n) + D_v\left[\frac{V_{j+1}^{n+1}-2V_j^{n+1}+V_{j-1}^{n+1}}{(\Delta x)^2}\right] + (D_s\xi(t)+E(t))\left[\frac{V_{j+1}^n-V_{j-1}^n}{2\Delta x}\right], \tag{5}$$

Table 1. Traveling wave propagation under different F and $T_{\rm in}$ with noise intensity $D_{\rm s} = 0$; symbol -1 indicates the system sends out only one wave; symbol $-$ indicates the system sends out traveling waves periodically but the waves disappear quickly; symbol $+$ indicates the system sends out traveling waves periodically and the waves can propagate persistently.

	$T_{\rm in} < 2.0$	$T_{\rm in} = 2.0$	$T_{\rm in} = 3.0$	$T_{\rm in} = 4.3$	$T_{\rm in} = 5.0$	$T_{\rm in} = 6.0$	$T_{\rm in} > 6.0$
$F < 10.0$	-1	-1	-1	$-$	$-$	$-$	$-$
$F = 10.0$	-1	-1	-1	-1	$+$	$-$	$-$
$F = 20.0$	-1	-1	-1	$+$	$+$	$-$	$-$
$F = 30.0$	-1	-1	$+$	$+$	$+$	$-$	$-$

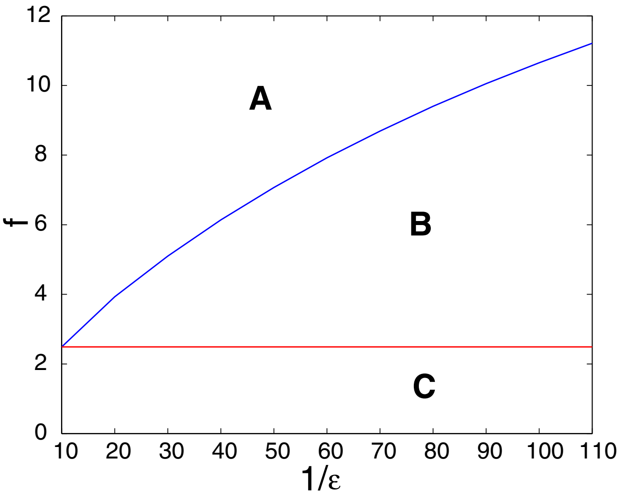

Fig. 1. (Color online) The phase diagram of ε versus f.

where $D_{\rm s}$, F, and $T_{\rm in}$ are the control parameters. Note that here the noise term $\xi(t)$ is Gaussian and white. This is a very reasonable assumption for internal noise, which represents many irrelevant degrees of freedom evolving over very short temporal and spatial scales. It has a probability density function with a normal distribution (also known as Gaussian distribution). In other words, the values that the noise can take on are Gaussian distributed.

3 Result

Extensive testing is performed through numerical simulations of the described model (3). The qualitative results are shown this section.

3.1 Sub-excitation and reversal phenomena

q, ϵ and f are parameters related to the BZ kinetics, determining the sub-excitation of the system. $q = 0.002$ [35]. First we simulate the dynamical system (3) without noise and periodic force to confirm the sub-excitable region. The result is shown in Figure 1. In region (A) no traveling waves are produced in the system; in region (B) traveling waves are sent out and propagate in the system; region (C) is not included in the excitable region. In the area between region (A) and (B), traveling waves are sent out but die away gradually, which is the so-called sub-excitable region, corresponding to the blue line in Figure 1. Here we set $\epsilon = 0.1$ and $f = 2.435$ to fix our system into in a sub-excitable region. When the parameters ϵ and f are deeply in the sub-excitable domain, the same results are observed by the numerical experiments, such as, $\epsilon = 0.025$, $f = 2.435$ and $\epsilon = 0.025$, $f = 4.0$.

It turns out that the influence of noise is rather important in the sub-excitable media. There is an extreme case for the system (3), that is, when the intensity of noise equals 0, i.e. $D_{\rm s} = 0$, and only the periodic force is present. The different types of wave propagation are summarized in the Table 1. Table 1 reveals that there exists critical input period $T^*_{\rm in}$ for each intensity of periodic force F ($F > 10.0$), so that the system creates traveling waves periodically and waves propagate persistently. If $F < 10.0$, the system can only send out one wave, and there is no critical input period $T^*_{\rm in}$; The system sends out traveling waves periodically but the waves disappear quickly when $T_{\rm in} > T^*_{\rm in}$ (see *Movie-1* for this case, where $D_{\rm s} = 0$, $T_{\rm in} = 8.0$, and $F = 10.0$); when $F = 10.0$, $T^*_{\rm in} = 5.0$; $F = 20.0$, $T^*_{\rm in} = 4.0, 5.0$; $F = 30.0$, $T^*_{\rm in} = 3.0, 4.0, 5.0$.

We next turn on the noise and periodic forces in system (3) and investigate their effects when the intensity of noise takes different values. Figure 2 shows that traveling waves propagate with different noise and periodic forces. The abscissa is the spatial location and the ordinate is the evolution of time. The white part indicates wave crests. As shown in Figure 2A, if only noise is present, traveling waves are produced irregularly but they can propagate stably [see *Movie-2*]. If periodic force and noise are both present, travelling waves are produced periodically and reversal phenomena appear, however traveling waves die out quickly when the intensity of noise is small [see *Movie-3*]. The travelling waves are produced periodically and they propagate stably, and reversal phenomena also appear when the intensity of noise is increased [see *Movie-4*], as shown in Figure 2C. In Figure 3, we show the phase diagram for reversal phenomena with respect to the $D_{\rm s}$–F parameter space, in which the reversal travelling waves die out quickly within region I but propagate persistently within region II. One can see that the reversal waves sensitively depend on the intensity of periodic forces and the noise intensities for the fixing T_{in}. For example (see Fig. 3), the emergence of reversal waves is a sensitive relationship before and after the certain critical value. For large F, the curve sharply decreases, otherwise it decreased slowly. So we can conclude from Figures 2

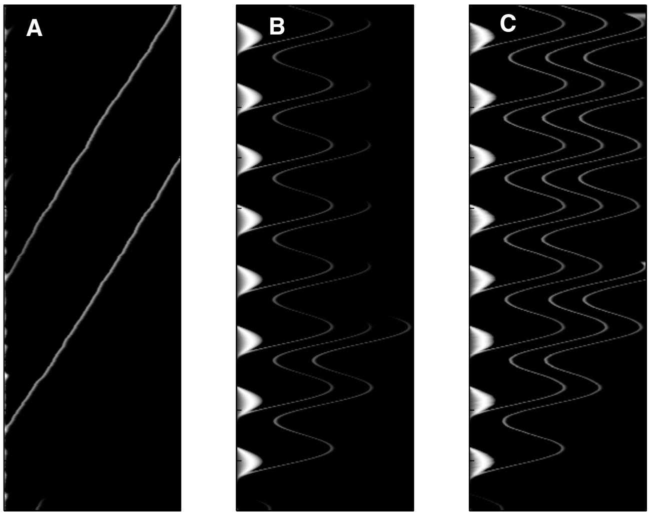

Fig. 2. (Color online) The spatiotemporal plot of the variable u for the system (3), where the ordinate is time evolution and the abscissa represents the spatial location. The white part indicates wave front. (A) Only noise present, $D_s = 14.0$, and $F = 0$ (see *Movie-2*, additional movies available from the Journal Website); (B) both noise and periodic forcing present, $D_s = 10.0$, $F = 10.0$, and $T_{in} = 8.0$ (see *Movie-3*, additional movies available from the Journal Website); (C) both noise and periodic forcing present, $D_s = 14.0$, $F = 10.0$, and $T_{in} = 8.0$ (see *Movie-4*, additional movies available from Journal Website).

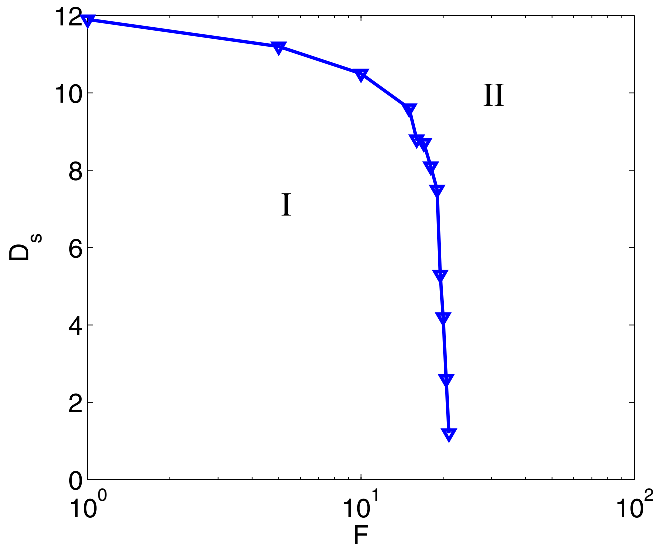

Fig. 3. (Color online) Phase diagram showing the reversal phenomena with respect to D_s–F parameter space. The parameters are the same as Figure 2, but $T_{in} = 8.0$. The reversal waves will die out quickly within region I and propagate persistently within region II. Note the log scale for T_{in}. (▼, the critical values from simulation results.)

and 3, that the stabilization of traveling waves are due to noise, but the periodicity owe to the function of periodic force. Periodical and stable traveling waves, as well as reversal phenomena, are produced in sub-excitable systems driven by noise and periodic force.

To characterize the relation between the periodic force and reversal phenomena, we give some qualitative explanations. Figure 4 shows the relationship between the propagation of traveling waves with the periodic force $E(t) = F\sin(\frac{2\pi}{T_{in}}t)$. The left window of Figure 4 is the periodic force whose abscissa is $E(t)$ and the right window represents the spatiotemporal plot of variable u whose abscissa is the spatial location. The left and right window share the same ordinate which is time. From Figure 4 we can easily observe that corresponding to each input period the system sends out a traveling wave, which is 1:1 frequency locking. The dotted line in the left window of Figure 4 separates out the positive and negative parts of the periodic force. We observe that if the periodic force is negative, traveling waves propagate forward, whereas if the periodic force is positive, traveling waves propagate backward in the opposite direction. Thus reversal phenomena appear.

For further investigation, we test several different kinds of periodic force for system (3). They are $E_\sqcap(t) = (-1)^{\lfloor \frac{2t}{T_{in}} \rfloor} F$ (the rectangle periodic force, here $\lfloor n \rfloor$ denotes the integer of n), $E_-(t) = -|F\sin(\frac{2\pi}{T_{in}}t)|$ and $E_+(t) = |F\sin(\frac{2\pi}{T_{in}}t)|$. When the periodic forcing is $E_\sqcap(t)$, travelling

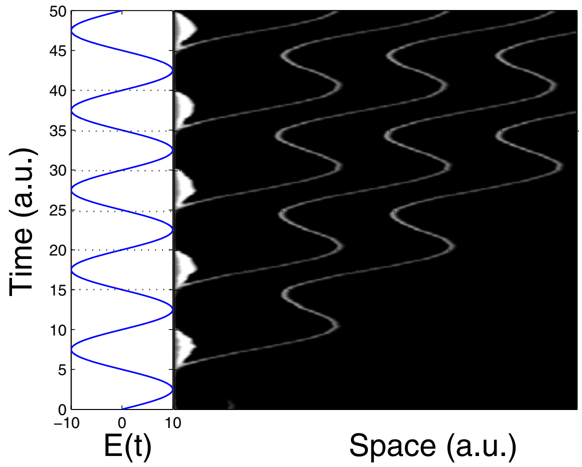

Fig. 4. (Color online) The propagation of traveling waves with periodic force $E(t) = F\sin(\frac{2\pi}{T_{in}}t)$. The ordinate is time evolution. For the left window the abscissa represents $E(t)$ and for the right the abscissa is space location. The left and right windows share the same ordinate which is time. The parameters are $D_s = 14.0$, $F = 10.0$, and $T_{in} = 8.0$.

waves are produced periodically and they propagate stably, and reversal phenomena appear (resembling Fig. 2C). When the periodic force is $E_-(t)$ or $E_+(t)$, there are only foundations formed but no travelling waves are sent out, as shown in Figure 5. So the reversal phenomenon is due to the alternation of the positive and negative values of the periodic force. If each value of the periodic force was positive (or negative), no travelling waves will be sent out and of course no reversal phenomena will appear.

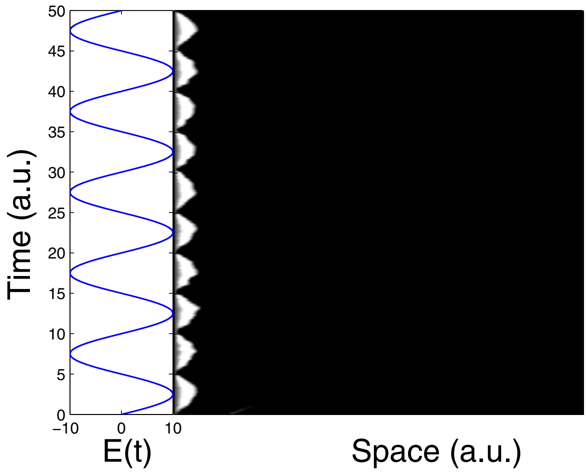

Fig. 5. (Color online) The propagation of travelling waves with periodic forcing $E(t) = E_-(t)$ (or $E(t) = E_+(t)$). The ordinate is time evolution. For the left window the abscissa represents $E(t)$ and for the right the abscissa is space location. The left and right windows share the same ordinate which is time. The parameters are $D_s = 14.0$, $F = 10.0$, and $T_{in} = 8.0$.

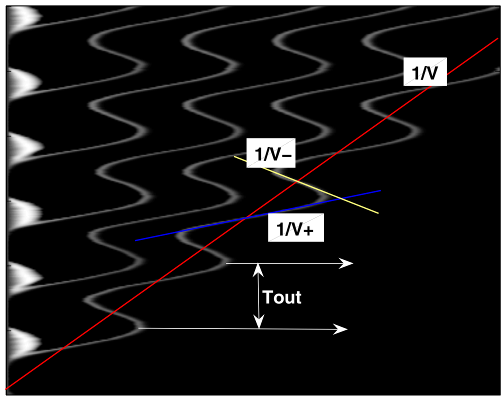

Fig. 6. (Color online) The sketch map of T_{out}, V, V_+, and V_- with $D_s = 14.0, F = 10.0, T_{in} = 8.0$.

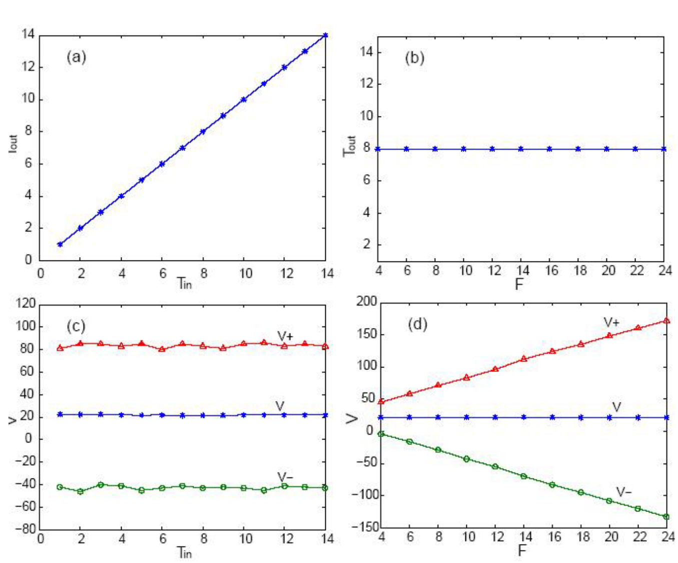

Fig. 7. (Color online) The effect of periodic force on the property of traveling waves, $D_s = 14.0$. (a) The plot of the output period with respect to the input period, $F = 10.0$. (b) The plot of the output period with respect to the intensity of periodic force, $T_{in} = 8.0$. (c) The plot of the three velocities of the traveling waves with respect to the input period, and $F = 10.0$. (d) The plot of the three velocities of the traveling waves with respect to the intensity of periodic force, $T_{in} = 8.0$.

3.2 The output period and the three velocities

In this section we focus on the effect of the periodic force $E(t) = F\sin(\frac{2\pi}{T_{in}}t)$ on the propagation of travelling waves. Here, the noise intensity D_s is fixed at 14.0. Furthermore, we define several quantities of the traveling waves. The output period T_{out} is defined as follows: T_i is the time interval between the ith wave and the $i+1$th wave. m waves are taken into account and the average value of them is T_{out}, where $T_{out} = \sum_{i=1}^{m} T_i/(m-1)$. The three velocities (the normal, the positive and the negative) of the traveling waves are defined as follows: when traveling waves propagate forward, there is a mean propagation velocity which is the positive velocity denoted by V_+. The mean velocity of the backward propagating waves is likewise denoted by V_-. In addition, V is defined here as the average velocity of the whole propagation of traveling wave. The output period T_{out}, the normal velocity V, the positive velocity V_+ and the negative velocity V_- are shown in Figure 6. The slopes of the red, blue and yellow lines in Figure 6 are $1/V$, $1/V_+$ and $1/V_-$, respectively.

Based on these definition, we now focus our simulations on the relationship between periodic force and the properties of travelling waves. The results are shown in Figure 7. Above all we interpret how the periodic force $E(t)$ affects the output period T_{out}. First, we fix $F = 10.0$ (the intensity of the periodic force) and study how T_{out} changes with T_{in} (the input period),as shown in Figure 7a. One observes that the output period increases linearly with the input period. Moreover, the output period is the same as the input period (the slope equals to 1). Second, we fix $T_{in} = 8.0$ and study the relation between T_{out} and F, as shown in Figure 7b, from which one observes that the output period is a constant and independent of the intensity of the periodic forcing. We can therefore draw a conclusion that the output period is the same as the input period and is independent of the intensity of the periodic force, under a fixed noise intensity.

Next we investigate the influence of the periodic force $E(t)$ on the three velocities of the travelling waves (the normal V, the positive V_+ and the negative V_-). First we fix the intensity of the period forcing $F = 10.0$ and see the changes of V, V_+, and V_- with the input period T_{in}, respectively. The results are shown in Figure 7c, from which we observe that the normal, positive and negative velocities approximate constant values, respectively, independent of the input period. Then we fix $T_{in} = 8.0$ and study the changes of V, V_+ and V_- with F (see Fig. 7d), which shows that the normal velocity is constant independent of the intensity of the periodic forcing; the positive velocity increases with the intensity of periodic force, while the negative velocity decreases with the intensity of periodic force. So we can conclude that the normal velocity is independent of the periodic force; the positive and negative velocities have the same trend with the intensity of periodic forces and do not depend on the input period.

It is natural to presume from the above results that the three velocities are interrelated. Through examination of the data, we obtain the relationship between the normal, positive and negative velocities, as follows

$$2V = V_+ + V_-, \tag{6}$$

that is,

$$V^* = V_+ - V = V - V_-. \tag{7}$$

So the positive velocity is the normal velocity plus V^*, and the negative velocity is the normal velocity minus V^*.

4 conclusion and discussion

In conclusion, noise and periodic force play a very important role on the production, propagation, and stability of the traveling waves in sub-excitable systems. Noise can induce traveling waves to propagate stably. It can also support wave propagation in sub-excitable [15,17,20] media due to a noise-induced transition [21,22,36]. In these studies, the media are static, and transport is governed by diffusion. In many systems, the media are not static, but subject to a motion, for example, when stirred by a flow, or by the oscillatory electric field (with period). This occurs especially in chemical reactions in a fluid environment. In this case, usually diffusive transport dominates only at small spatial scales while mixing due to the flow in much faster at large scale. In this paper, we investigate the sub-excitable system using a simplified Oregonator model, and the propagation of traveling waves in the presence of both noise and periodic force. Depending on noise and the periodic forcing we have observed different types of wave propagation (or its disappearance). Moreover, the reversal phenomena is observed in this system based on the numerical experiments in one-dimensional space. We give qualitative explanations to how reversal phenomena appear, which turns out to be due to the periodic force. The output period and three velocities (the normal, positive and negative) of the traveling waves are defined and their relationship with the periodic forcing are also studied in sub-excitable system with a fixed intensity of noise.

The periodic force can make periodically produced traveling waves and reversal phenomena appear. The reversal phenomenon results from the alternation of the positive and negative values of the periodic forcing and noise. For the special case, we show the phase diagram for the reversal phenomena with respect to D_s–F parameter space, from which one can see that the reversal waves sensitively depend on the intensity of periodic forces and the noise intensities for the fixing T_{in}. Finally, we examine the effect of periodic force on the sub-excitable system with a fixed noise intensity $D_s = 14.0$. Under such a condition, the influence of periodic forces on the three different kinds of velocities are also determined. The relation among the three velocities is $2V = V_+ + V_-$. In reference [20], the authors studied sub-excitable medium of a Belousov-Zhabotinsky (BZ) reaction subjected to Gaussian white noise in experiments. They observed that at an optimal level of noise the wave sources of excited traveling waves become synchronous, as though there exists a long distance spatial correlation.

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References

1. W. Horsthemke, R. Lefever, *Noise-Induced Transi-tions: Theory and Applications in Physics, Chemistry and Biology* (Springer-Verlag, Berlin, 1984)
2. R. Mankin, T. Laas, A. Sauga, A. Ainsaar, E. Reiter, Phys. Rev. E **74**, 021101 (2006)
3. S. Aumaître, K. Mallick, F. Pétrélis, J. Stat. Mech. **2007**, P07016 (2007)
4. L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998)
5. H. Zhonghuai, Y. Lingfa, X. Zuo, X. Houwen, Phys. Rev. Lett. **81**, 2854 (1998)
6. K. Wiesenteld, F. Moss, Nature (London) **373**, 33 (1995)
7. P. Jung, G. Mayer-Kress, Phys. Rev. Lett. **74**, 2130 (1995)
8. J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Löcher, W.L. Ditto, Phys. Rev. Lett. **81**, 5048 (1998)
9. F. Marchesoni, L. Gammaitoni, A.R. Bulsara, Phys. Rev. Lett. **76**, 2609 (1996)
10. M. Marchi, F. Marchesoni, L. Gammaitoni, E. Menichella-Saetta, S. Santucci, Phys. Rev. E **54**, 3479 (1996)
11. T. Leiber, F. Marchesoni, H. Risken, Phys. Rev. A **38**, 983 (1988)
12. M. Feingold, D.L. Gonzalez, O. Piro, H. Viturro, Phys. Rev. A **37**, 4060 (1988)
13. E. Meron, Phys. Rep. **218**, 1 (1992)
14. A.S. Mikhailov, *Foundations of Synergetics I: Distributed Active Systems* (Springer, New York, 1990)
15. S. Kádár, J. Wang, K. Showalter, Nature **391**, 770 (1998)
16. J. Wang, S. Kádár, P. Jung, K. Showalter, Phys. Rev. Lett. **82**, 855 (1999)
17. H. Hempel, L. Schimansky-Geier, J. García-Ojalvo, Phys. Rev. Lett. **82**, 3713 (1999)
18. A.N. Zakin, A.M. Zhabotinsky, Nature **225**, 535 (1970)
19. R.J. Field, E. Koros, R.M. Noyes, J. Am. Chem. Soc. **94**, 8649 (1972)
20. L.Q. Zhou, X. Jia, Q. Ouyang, Phys. Rev. Lett. **88**, 138301 (2002)
21. S. Alonso, I. Sendiña-Nadal, V. Pérez-Muñuzuri, J.M. Sancho, F. Sagués, Phys. Rev. Lett. **87**, 078302 (2001)

22. J. Garcia-Ojalvo, J.M. Sancho, *Noise in spatially extended systems* (Springer, New York, 1999)
23. D.S. Dean, I.T. Drummond, R.R. Horgan, J. Stat. Mech. **2007**, P07013 (2007)
24. V. Krinsky, E. Hamm, V. Voignier, Phys. Rev. Lett. **76**, 3854 (1996)
25. Z. Neufeld, I.Z. Kiss, C. Zhou, J. Kurths, Phys. Rev. Lett. **91**, 084101 (2003)
26. Z. Neufeld, C. López, E. Hernández-García, O. Piro, Phys. Rev. E **66**, 066208 (2002)
27. C. Zhou, J. Kurths, New J. Phys. **7**, 18 (2005)
28. C. Zhou, J. Kurths, Z. Neufeld, I.Z. Kiss, Phys. Rev. Lett. **91**, 150601 (2003)
29. J.J. Tyson, P.C. Fife, J. Chem. Phys. **73**, 2224 (1980)
30. R.A. Albanese, in *Inverse Problems in Wave Propagation*, edited by G. Chavent, G. Papanicolaou, P. Sacks, W. Symes (Springer-Verlag, 1997)
31. W. Jahnke, A.T. Winfree, Inter. J. Bifur. Chaos **1**, 445 (1991)
32. D. Higham, SIAM Rev. **43**, (2001)
33. W. Jahnke, W.E. Skaggs, A.T. Winfree, J. Phys. Chem. **93**, 740 (1989)
34. A.T. Winfree, W. Jahnke, J. Phys. Chem. **93**, 2823 (1989)
35. J.J. Tyson, *Oscillations and traveling waves in chemical systems* (Wiley, N.Y., 1985)
36. F. Sagués, J.M. Sancho, J. Garcia-Ojalvo, Rev. Mode. Phys. **79**, 829 (2007)